

Assignment for class XII

General Directions For Students : Whatever be the notes provided , everything must be copied in the maths copy and then do the homework in the same copy

Topic : Implicit Differentiation

Chapter 5: Exercise 5.8

- An Implicit equation is an equation which is not in the form of $y = f(x)$
- It consists of two variables x and y which cannot be separated.
- Implicit functions are differentiated by using "chain rule " in combination with " product and quotient rule "
- When we differentiate y we write $\frac{dy}{dx}$ with the derivative. (i.e. $\frac{d}{dx}(y^2) = 2y \frac{dy}{dx}$.)
- To find the derivative of the product of x and y i.e. $\frac{d}{dx}(xy)$ we use product rule

Q.3.ii) Find $\frac{dy}{dx}$ if $x^3 + x^2y + xy^2 + y^3 = 81$

$$\text{solution. } \frac{d}{dx}(x^3) + x^2 \frac{d}{dx}(y) + y \frac{d}{dx}(x^2) + x \frac{d}{dx}(y^2) + y^2 \frac{d}{dx}(x) + \frac{d}{dx}(y^3) = \frac{d}{dx}(81)$$

$$\Rightarrow 3x^2 + x^2 \frac{dy}{dx} + 2xy + x(2y) \frac{dy}{dx} + y^2 + 3y^2 \frac{dy}{dx} = 0$$

$$\Rightarrow (x^2 + 2xy + 3y^2) \frac{dy}{dx} + (3x^2 + 2xy + y^2) = 0$$

$$\Rightarrow (x^2 + 2xy + 3y^2) \frac{dy}{dx} = -(3x^2 + 2xy + y^2)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{3x^2 + 2xy + y^2}{x^2 + 2xy + 3y^2}$$

Q.5.ii) Find $\frac{dy}{dx}$ if $\sin(x+y) = \frac{2}{3}$

$$\text{If } \sin(x+y) = \frac{2}{3} \text{ then } x+y = \sin^{-1} \frac{2}{3}$$

$$\Rightarrow \frac{d}{dx}(x+y) = \frac{d}{dx}(\sin^{-1} \frac{2}{3})$$

$$\Rightarrow 1 + \frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -1$$

Q.6. If $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 2$, find $\frac{dy}{dx}$ at $(1,1)$

$$\frac{d}{dy}(x^{\frac{2}{3}} + y^{\frac{2}{3}}) = \frac{d}{dy}(2)$$

$$\Rightarrow \frac{d}{dy}(x^{\frac{2}{3}}) + \frac{d}{dy}(y^{\frac{2}{3}}) = 0$$

$$\Rightarrow \frac{2}{3}x^{-\frac{1}{3}} + \frac{2}{3}y^{-\frac{1}{3}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\frac{2}{3}x^{-\frac{1}{3}}}{\frac{2}{3}y^{-\frac{1}{3}}} = \frac{1^{-\frac{1}{3}}}{1^{-\frac{1}{3}}} = -1 \quad (\because x=1, y=1)$$

Q.8vi) Find $\frac{dy}{dx}$, when $y = \tan(x+y)$

$$\frac{dy}{dx} = \frac{d}{d(x+y)}(\tan(x+y)) \frac{d}{dx}(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \sec^2(x+y) \frac{d}{dx}(x+y)$$

$$\Rightarrow \frac{dy}{dx} = \sec^2(x+y) \left(1 + \frac{dy}{dx}\right)$$

$$\Rightarrow \frac{dy}{dx} = \sec^2(x+y) + \sec^2(x+y) \frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} - \sec^2(x+y) \frac{dy}{dx} = \sec^2(x+y)$$

$$\Rightarrow (1 - \sec^2(x+y)) \frac{dy}{dx} = \sec^2(x+y)$$

$$\Rightarrow -\tan(x+y)^2 \frac{dy}{dx} = \sec^2(x+y)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sec^2(x+y)}{\tan^2(x+y)} = -\operatorname{cosec}^2(x+y)$$

$$\Rightarrow \frac{dy}{dx} = -\operatorname{cosec}^2(x+y)$$

Homework:

Q.2.i), 4.i),5.i)Q.7.ii),Q.8.v),8.viii)Q.10.ii).Q.11.iii)

Solution of following questions are discussed in the video link provided to you with this assignment:

Q7.i),Q.8.iii),vii),Q.9),Q.10i),iii),Q.11i)

